## Exercises- Propositional Logic

**Exercise 1**

Check the following properties for (’*nor*’), (’*nand*’) and (’*xor*’) connectives using the truth table method.

1. associativity of ’’ connective: ;
2. associativity of ’’ connective: ;
3. associativity of ’**’ connective: ;
4. distribution of ’’ connective over ’’ connective:

;

1. distribution of ’’ connective over ’’ connective:

;

1. De Morgan’s laws for ’’ and ’’:

 and ;

1.  and .
2.  and ;

**Exercise 2**

Using the truth table method decide what kind of formula (consistent, inconsistent, tautology, contingent) is . Write all the models and anti-models of .

|  |  |
| --- | --- |
| 1. ; | 1. ; |
| 1. ; | 1. ; |
| 1. ; | 1. ; |
| 1. ; | 1. . |

**Exercise 3**

Using the truth table method, check whether the following logical consequences hold:

|  |  |
| --- | --- |
| 1. ; | 1. ; |
| 1. ; | 1. ; |
| 1. ; | 1. ; |
| 1. ; | 1. . |

**Exercise 4.**

Prove that the following formulas are tautologies using the truth table method.

1. the left-distribution of ’’ over ’’: ;
2. the permutation of the premises law: ;
3. the reunion of the premises law:;
4. the separation of the premises law: ;
5. the left-distribution of ’’ over ’’: ;
6. the left-distribution of ’’ over ’’:;
7. the left-distribution of ’’ over ’’:;
8. the left-distribution of ’’ over ’’:.

**Exercise 5**

Transform the formulas  into their equivalent conjunctive and disjunctive normal forms. Using one of these forms prove that  are valid formulas in propositional logic.

1. ;
2. ;
3. ;
4. ;
5. ;
6. ;
7. ;
8. .

**Exercise 6**

Using the appropriate normal form write all the models of the following formulas:

1. ;
2. ;
3. ;
4. ;
5. ;
6. ;
7. ;
8. .

**Exercise 7**

Using the appropriate normal form, prove that the following formulas are inconsistent:

1. ;
2. ;
3. ;
4. ;
5. ;
6. ;
7. ;
8. .

**Exercise 8**

Write all the anti-models of the following formulas using CNF.

1. ;
2. ;
3. ;
4. ;
5. ;
6. ;
7. ;
8. .

**Exercise 9**

Using the definition of deduction, prove the following deductions:

|  |  |
| --- | --- |
| 1. ; | 1. ; |
| 1. ; | 1. ; |
| 1. ; | 1. ; |
| 1. ; | 1. . |

**Exercise 10**

Prove the following theorems using the theorem of deduction and its reverse.

1. ;
2. ;
3. ;
4. ;
5. ;
6. ;
7. ;
8. .

**Exercise 11**

Using the theorem of deduction and its reverse prove that:

1. ;
2. ;
3. ;
4. ;
5. ;
6. ;
7. ;
8. .

**Exercise 12**

H1: It is not sunny this afternoon and it is colder than yesterday.

H2: We will go swimming only if it is sunny.

H3: If we do not go swimming, then we will take a canoe trip.

H4: If we take a canoe trip, then we will be home by sunset.

C: We will be home by sunset.

Is C deducible from the set of hypotheses {H1,H2,H3,H4}?

If yes, build its deduction.